

Quant Concepts: Session 1 + 2 – Statistics, Numbers, Inequalities, Mods (Absolute Values)**The trickiest and the most important topics for GMAT Quant****Mean (Average)**

1. The arithmetic mean of a set of values is the quantity commonly called the mean or the average. Given a set of samples $\{x_i\}$, the arithmetic mean is $\Sigma X / N$.
2. If each one of the given numbers is increased (or decreased or multiplied or divided) by K , their average is increased (or decreased or multiplied or divided) by K .
3. If there are r series of observations N_1, N_2, \dots, N_r , the mean M of the whole series is related to the mean M_1, M_2, \dots of the component series by the equation $NM = N_1M_1 + N_2M_2 + \dots + N_rM_r$, where $N = N_1 + N_2 + \dots + N_r$.
4. The sum of first " n " natural numbers is given by $n(n + 1)/2$.
5. For consecutive integers or for equally spaced numbers (AP), Mean = (First term + Last term) / 2 = Median
6. Count of consecutive numbers inclusive = last term – first term + 1, Example 9 to 15, total = 7. Count of consecutive numbers exclusive (terms greater than x but less than y) = last term – first term – 1. Example: 9 to 15, total = 5
7. If the average of a few consecutive integers is 0, then either all numbers are zero or there will be an odd number of integers.
8. The average of an odd number of consecutive integers is an integer and the average of an even number of consecutive integers is a non-integer.
9. If in a set of numbers, the average = the highest or the lowest number, all the numbers will have to be equal.
10. Concept of Mixed Average (weighted average):

$$\frac{N1}{N2} = \frac{M2 - M}{M - M1} = \frac{D2}{D1}$$

Problems

1. In a work force, the employees are either managers or directors. What is the percentage of directors?
(1) the average salary for manager is \$5,000 less than the total average salary.
(2) the average salary for directors is \$15,000 more than the total average salary.
2. A group of men and women gathered to compete in a marathon. Before the competition, each competitor was weighed and the average weight of the female competitors was found to be 120 lbs. What percentage of the competitors were women?
(1) The average weight of the men was 150 lb.
(2) The average weight of the entire group was twice as close to the average weight of the men as it was to the average weight of the women.
3. Committee X and Committee Y, which have no common members, will combine to form Committee Z. Does Committee X have more members than Committee Y?
(1) The average (arithmetic mean) age of the members of Committee X is 25.7 years and the average age of the members of Committee Y is 29.3 years.
(2) The average (arithmetic mean) age of the members of Committee Z will be 26.6 years
4. Of the 1400 College teachers surveyed 42% said they considered engaging in research an essential goal. How many of the college teachers surveyed were women?
(1) In the survey 36% of the men and 50% of the women said that they considered engaging in research an essential goal.
(2) In the survey, 288 men said they considered engaging in research an essential goal.
5. In a certain senior class, 72% of the male students and 80% of the female students have applied to college. What fraction of the students in the senior class is male?
(1) There are 840 students in the senior class (2) 75% of the students in the senior class have applied to college

6. At a certain company the average number years of experience is 9.8 years for males and 9.1 years for females. What is the ratio of the number of the company's male employees to the number of the company's female employees?
 - (1) There are 52 male employees at the company.
 - (2) The average number of years of experience for the company's males and females combined is 9.3 years.
7. During a sale, a clothing store sold each shirt at a price of \$15 and each sweater at a price of \$25. Did the store sell more sweaters than shirts during the sale?
 - (1) The average (arithmetic mean) of the prices of all the shirts and sweaters that the store sold during the sale was \$21.
 - (2) The total of the prices of all the shirts and sweaters that the store sold during the sale was \$420
8. A convenience store currently stocks 48 bottles of mineral water. The bottles have two sizes of either 20 or 40 ounces each. The average volume per bottle the store currently has in stock is 35 ounces. How many 40 ounce bottles must be sold for the average volume per bottle to be reduced to 25 ounces if no 20 ounce bottles are sold?

10	20	30	32	34
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Median:

- Median is the middle value or the average of two middle values when the values are arranged in an order, either ascending or descending.
- If there are odd number of observations, median is directly the middle number.
- If there are even number of observations, median is the average of the two middle numbers.
- For consecutive integers or for equally spaced numbers (AP), Median = (First term + Last term) / 2. So, Median = Mean in this case.
- Median is the 50th percentile.

Problems

1. Which of the following could be the median of a set consisting of 6 different primes?

(A) 2	(B) 3	(C) 9.5	(D) 12.5	(E) 39
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2. What is the median number of employees assigned per project for the projects at Company Z?
 - (1) 25 percent of the projects at Company Z have 4 or more employees assigned to each project.
 - (2) 35 percent of the projects at Company Z have 2 or fewer employees assigned to each project.
3. Tom, Jane, and Sue each purchased a new house. The average (arithmetic mean) price of the three houses was \$120,000. What was the median price of the three houses?
 - (1) The price of Tom's house was \$110,000.
 - (2) The price of Jane's house was \$120,000.
4. What is the median value of the set R , if for every term in the set, $R_n = R_{n-1} + 3$?
 - (1) The first term of set R is 15.
 - (2) The mean of set R is 36.
5. Peter, Paul, and Mary each received a passing score on his/her history midterm. The average (arithmetic mean) of the three scores was 78. What was the median of the three scores?
 - (1) Peter scored a 73 on his exam.
 - (2) Mary scored a 78 on her exam.
6. Last month 15 homes were sold in Town X. The average (arithmetic mean) sale price of the homes was \$150,000 and the median sale price was \$130,000. Which of the following statement must be true?
 - I. at least one of the homes was sold for more than \$165,000
 - II. at least one of the homes was sold for more than \$130,000 and less than \$150,000
 - III. at least one of the homes was sold for less than \$130,000

a) I only	b) II only	c) III only	d) I and II	e) I and III
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Score	Number and Interval of Scores
50–59	2
60–69	10
70–79	16
80–89	27
90–99	18

The table above shows the distribution of test scores for a group of management trainees, which score interval contains the median of the 73 scores?

- A. 60–69 B. 70–79 C. 80–89 D. 90–99 E. Can't get
8. If x and y are unknown positive integers, is the mean of the set $\{6, 7, 1, 5, x, y\}$ greater than the median of the set?
 (1) $x + y = 7$ (2) $x - y = 3$
9. Set S consists of five consecutive integers, and set T consists of seven consecutive integers. Is the median of the numbers in set S equal to the median of the numbers in set T ?
 (1) The median of the numbers in Set S is 0.
 (2) The sum of the numbers in set S is equal to the sum of the numbers in set T .
10. The sum of the integers in list S is the same as the sum of the integers in list T . Does S contain more integers than T ?
 (1) The average (arithmetic mean) of the integers in S is less than the average of the integers in T .
 (2) The median of the integers in S is greater than the median of the integers in T .
11. In a sequence that contains 15 terms from a_1 to a_{15} , $a_n = a_{n-1} + k$, where k is a non-zero constant. How many terms in the sequence are greater than 10?
 (1) $a_1 = 24$ (2) $a_8 = 10$

Range

Range is defined as the difference between the two extreme observations of the distribution.

Range = $X_{\max} - X_{\min}$. If Range = 0, all the observations are equal. Range ≥ 0 always. Never negative.

1. Set A consists of 8 distinct prime numbers. If x is equal to the range of set A and y is equal to the median of set A , is the product xy even?
 (1) The smallest integer in the set is 5. (2) The largest integer in the set is 101.
2. If S is a finite set of consecutive even numbers, is the median of S an odd number?
 (1) The mean of set S is an even number. (2) The range of set S is divisible by 6.
3. A set of 15 different integers have a range of 25 and a median of 25. What is greatest possible integer that could be in this set?
 32 37 40 43 50
4. Set R contains five numbers that have an average value of 55. If the median of the set is equal to the mean, and the largest number in the set is equal to 20 more than three times the smallest number, what is the largest possible range for the numbers in the set?
 1) 78 2) 77 3) 66 4) 55 5) 52

Standard deviation:

- It is defined as positive square root of the A.M. of the squares of the deviations of the given observations from their A.M.
- If X_1, X_2, \dots, X_N is a set of N observations then its standard deviation is given by Standard Deviation

$$\sigma = \sqrt{\frac{\sum (x - \bar{x})^2}{N}}$$
- It is a measure of how much each value varies from the mean of all the values.
- Less SD implies more consistency, less variation, less spread, more compactness AND vice versa.
- If SD = 0, all the observations are equal.
- Range is always greater than SD, except when all observations are equal, when both are equal to 0.
- The square of SD is called Variance.

Change in respective statistical parameters:

	Addition	Subtraction	Sign Change	Multiplication	Division
Mean	Change	Change	Change	Change	Change
Median	Change	Change	Change	Change	Change
Range	NO Change	NO Change	NO Change	Change	Change
SD	NO Change	NO Change	NO Change	Change	Change

Problems

- I. 72, 73, 74, 75, 76 II. 74, 74, 74, 74, 74 III. 62, 74, 74, 74, 89
The data sets I, II, and III above are ordered from greatest standard deviation to least standard deviation in which of the following?
(A) I, II, III (B) I, III, II (C) II, III, I (D) III, I, II (E) III, II, I
- Set A consists of all prime numbers between 10 and 25; Set B consists of consecutive even integers, and set C consists of consecutive multiples of 7. If all the three sets have an equal number of terms, which of the following represents the ranking of these sets in an ascending order of the standard deviation?
(A) C, A, B (B) A, B, C (C) C, B, A (D) B, C, A (E) B, A, C
- Set A consists of all even integers between 2 and 100, inclusive. Set X is derived by reducing each term in set A by 50, set Y is derived by multiplying each term in set A by 1.5, and set Z is derived by dividing each term in set A by -4 . Which of the following represents the ranking of the three sets in descending order of standard deviation?
(A) X, Y, Z (B) X, Z, Y (C) Y, Z, X (D) Y, X, Z (E) Z, Y, X
- If M is a negative integer and K is a positive integer, which of the following could be the standard deviation of a set $\{-7, -5, -3, M, 0, 1, 3, K, 7\}$?
I. -1.5 II. -2 III. 0
(A) I only (B) II only (C) III only (D) I and III only (E) None
- Sets A, B and C are shown below. If number 100 is included in each of these sets, which of the following represents the correct ordering of the sets in terms of the absolute increase in their standard deviation, from largest to smallest?
A $\{30, 50, 70, 90, 110\}$, B $\{-20, -10, 0, 10, 20\}$, C $\{30, 35, 40, 45, 50\}$
(A) A, C, B (B) A, B, C (C) C, A, B (D) B, A, C (E) B, C, A
- If the mean of a data set is 75 and the standard deviation is 10, what is the range of scores that fall within one standard deviation of the mean?
- The mean score of a class on a test was 60 and the standard deviation was 15. If Elena's score was within 2 standard deviations of the mean, what is the lowest score she could have received?
- If $y = ax + b$, and if the standard deviation of x series is 'S', what is the standard deviation of y series?
- Let Set $T = \{2, 4, 5, 7\}$. Which of the following values, if added to Set T, would most increase the standard deviation of Set T? 1 3 6 8 14
- What is the standard deviation of Q, a set of consecutive integers?
(1) Q has 21 members. (2) The median value of set Q is 20.
- A certain list of 100 data has an average of 6 and a standard deviation of d, where d is positive. Which of the following pairs of data, when added to the list, must result in a list of 102 data with standard deviation less than d? A. -6 and 0 B. 0 and 0 C. 0 and 6 D. 0 and 12 E. 6 and 6
- During an experiment some water was removed from each of the 6 tanks. If the standard deviation of the volumes of the water at the beginning of the experiment was 10 gallons, what was the standard deviation of the volumes of the water after the experiment?
(1) For each tank 30% of the volume of the water that was in the tank before the beginning of the experiment was removed during the experiment
(2) The average (mean) volume of water in the tanks at the end of the experiment was 63 gallons.

Quant Session 2 – Numbers / Inequalities / Absolute Values (Modulus) / General Algebra

(This is the most important topic for GMAT Quant)

1. If x , y , and z are integers and $xy + z$ is an odd integer, is x an even integer?
(1) $xy + xz$ is an even integer (2) $y + xz$ is an odd integer
2. If w , x , y and z are integers such that w/x and y/z are integers, is $w/x + y/z$ odd?
(1) $wx + yz$ is odd (2) $wz + yx$ is odd
3. The integers m and p are such that $2 < m < p$, and m is not a factor of p . If r is the remainder when p is divided by m , is $r > 1$?
(1) the greatest common factor of m and p is 2 (2) the least common multiple of m and p is 30
4. If the integer n is greater than 1, is n equal to 2?
(1) n has exactly two positive factors (2) The difference between any two distinct positive factors is odd.
5. The function f is defined for all positive integers n by the following rule: $f(n)$ is the number of positive integers each of which is less than n and also has no positive factor in common with n other than 1. If p is a prime number then $f(p) = ?$
 $p - 1$ $p - 2$ $(p + 1) / 2$ $(p - 1) / 2$ 2
6. For every positive even integer n , the function $h(n)$ is defined to be the product of all the even integers from 2 to n , inclusive. If p is the smallest prime factor of $h(100) + 1$, then p is
A. between 2 and 10 B. between 10 and 20 C. between 20 and 30
D. between 30 and 40 E. greater than 40
7. Is the integer n odd?
(1) n is divisible by 3 (2) $2n$ is divisible by twice as many positive integers as n
8. If d is a positive integer, f is the product of the first 30 positive integers, what is the value of d ?
(1) 10^d is a factor of f (2) $d > 6$
9. The product of the units digit, the tens digit, and the hundreds digit of the positive integer m is 96. What is the unit's digit of m ?
(1) m is odd (2) The hundreds digit of m is 8
10. How many different prime numbers are factors of the positive integer n ?
(1) four different prime numbers are factors of $2n$
(2) four different prime numbers are factors of n^2 .
11. Does the integer k have a factor p such that $1 < p < k$?
(1) $k > 4!$ (2) $13! + 2 \leq k \leq 13! + 13$.
12. The positive integer k has exactly two positive prime factors, 3 and 7. If K has a total of 6 positive factors, including 1 and k , what is the value of K ?
(1) 3^2 is a factor of k (2) 7^2 is NOT a factor of k
13. If n and t are positive integers, what is the greatest prime factor of the product nt ?
(1) The greatest common factor of n and t is 5.
(2) The least common multiple of n and t is 105.
14. If t is a positive integer and r is the remainder when $t^2 + 5t + 6$ is divided by 7, what is the value of r ?
(1) when t is divided by 7, the remainder is 6 (2) when t^2 is divided by 7, the remainder is 1
15. If p is a positive odd integer, what is the remainder when p is divided by 4?
(1) When p is divided by 8, the remainder is 5.
(2) p is the sum of the squares of two positive integers.

16. If p , x , and y are positive integers, y is odd, and $p = x^2 + y^2$, is x divisible by 4?
 (1) When p is divided by 8, the remainder is 5. (2) $x - y = 3$.
17. If n and m are positive integers, what is the remainder when $3^{4n+2} + m$ is divided by 10?
 (1) $n = 2$ (2) $m = 1$
18. If n is a positive integer and r is the remainder when $(n - 1)(n + 1)$ is divided by 24, what is the value of r ?
 (1) n is not divisible by 2 (2) n is not divisible by 3
19. If N is a positive integer, is $(N^3 - N)$ divisible by 4?
 (1) $n = 2k + 1$, where K is an integer. (2) $n^2 + n$ is divisible by 6
20. If m and r are two numbers on a number line, what is the value of r ?
 (1) The distance between r and 0 is 3 times the distance between m and 0
 (2) 12 is halfway between m and r
21. On the number line shown, is zero halfway between r and s ? $\text{---}r\text{---}s\text{---}t\text{---}$
 (1) s is right to the zero
 (2) the distance between t and r is the same as the distance between t and $(-s)$
22. If s and t are two different numbers on the number line, is $s + t = 0$?
 (1) The distance between s and 0 is the same as the distance between t and 0
 (2) 0 is between s and t
23. If each term in the sum $a_1 + a_2 + \dots + a_n$ is either 7 or 77 and the sum equals 350, which of the following could be the value of n ?
 38 39 40 41 42
24. $2 + 2 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6 + 2^7 + 2^8$?
 2^9 2^{10} 2^{16} 2^{35} 2^{37}
25. A certain list contains several different integers. Is the product of the integers in the list positive?
 (1) The product of the greatest and the smallest of the integers in the list is positive
 (2) There is an even number of integers in the list
26. If there are more than two numbers in a certain list, is each of the numbers in the list equal to 0?
 (1) The product of any two numbers in the list is equal to 0.
 (2) The sum of any two numbers in the list is equal to 0.
27. If the operation \wedge is one of the four arithmetic operations addition, subtraction, multiplication, and division, is $(6 \wedge 2) \wedge 4 = 6 \wedge (2 \wedge 4)$?
 (1) $3 \wedge 2 > 3$ (2) $3 \wedge 1 = 3$
28. If n and P are integers, is $p > 0$? (1) $n + 1 > 0$ (2) $np > 0$
29. Is $x^4 + y^4 > z^4$? (1) $x^2 + y^2 > z^2$ (2) $x + y > z$
30. A certain jar contains only "b" black marbles, "w" white marbles and "r" red marbles. If one marble is to be chosen at random from the jar, is the probability that the marble chosen is red greater than the probability that the marble chosen will be white?
 (1) $r / (b + w) > w / (b + r)$ (2) $b - w > r$
31. If X and Y are positive, which of the following must be greater than $1 / \sqrt{(x + y)}$?
 I. $[\sqrt{(X+Y)}] / 2x$ II. $[(\sqrt{X}) + (\sqrt{Y})] / (x + y)$ III. $[(\sqrt{X}) - (\sqrt{Y})] / (x + y)$
 A. None B. I only C. II only D. I & III E. II & III
32. If $y \geq 0$, what is the value of x ? (1) $|x - 3| \geq y$ (2) $|x - 3| \leq -y$
33. Is $|x| = y - z$? (1) $x + y = z$ (2) $x < 0$
34. If $m > 0$ and $n > 0$, is $(m + x) / (n + x) > m / n$? (1) $m < n$ (2) $x > 0$

35. If x is positive which of the following could be correct ordering of $1/x$, $2x$, x^2 ?
 I. $x^2 < 2x < 1/x$ II. $x^2 < 1/x < 2x$ III. $2x < x^2 < 1/x$
 A. None B. I Only C. III Only D. I and II only E. I, II & III
36. If x is not equal to $-y$, is $(x - y) / (x + y) > 1$? (1) $x > 0$ (2) $y < 0$
37. If k is not equal to 0, 1, or -1 , is $1/k > 0$? (1) $1 / (k - 1) > 0$ (2) $1 / (k + 1) > 0$
38. If $w + x < 0$, is $w - y > 0$? (1) $x + y < 0$ (2) $y < x < w$
39. If $x < 0$, then $\sqrt{-x \times |x|}$ is
 A. $-x$ B. -1 C. 1 D. x E. \sqrt{x}
40. Is $\sqrt{(x-5)^2} = 5-x$? (1) $-x |x| > 0$ (2) $5 - x > 0$
41. Is $\sqrt{(x-3)^2} = 3-x$? (1) $x \neq 3$ (2) $-x |x| > 0$
42. Is $1 / (a-b) < (b-a)$? (1) $a < b$ (2) $1 < |a-b|$
43. Is $|x| < 1$? (1) $|x + 1| = 2|x - 1|$ (2) $|x - 3| > 0$
44. If $x > y^2 > Z^4$, which of the following statements could be true?
 I. $x > y > z$ II. $z > y > x$ III. $x > z > y$
 A. I only B. I and II only C. I and III only D. II and III only E. I, II, and III
45. If p , q , r , and s are consecutive integers, with $p < q < r < s$, is $pr < qs$?
 (1) $pq < rs$ (2) $ps < qr$
46. If y is an integer and $y = |x| + x$, is $y=0$? (1) $x < 0$ (2) $y < 1$
47. Is $M + Z > 0$ (1) $M - 3Z > 0$ (2) $4Z - M > 0$
48. If $zy < xy < 0$, is $|x - z| + |x| = |z|$ (1) $z < x$ (2) $y > 0$
49. If $Z^n = 1$, what is the value of Z ? (1) n is a non zero integer (2) $Z > 0$
50. If $a < y < z < b$, is $|y - a| < |y - b|$? (1) $|z - a| < |z - b|$ (2) $|y - a| < |z - b|$
51. The numbers x and y are not integers. The value of x is closest to which integer?
 (1) 4 is the integer that is closest to $x + y$ (2) 1 is the integer that is closest to $x - y$
52. Are positive integers P and Q both greater than N ?
 (1) $P - Q$ is greater than N (2) $Q > P$
53. Is Z an integer? (1) Z^3 is an integer (2) $3Z$ is an integer
54. Are x and y both positive (1) $2x - 2y = 1$ (2) $x/y > 1$
55. Is $mx + ky > kx + my$? (1) $m > k$ (2) $x > y$
56. If 500 is the multiple of 100 that is closest to X and 400 is the multiple of 100 closest to Y , then which multiple of 100 closest to $X + Y$?
 (1) $X < 500$ (2) $Y < 400$
57. Is $w > 1$? (1) $w + 2 > 0$ (2) $w^2 > 1$
58. $1 / p > [r / (r^2 + 2)]$ (1) $p = r$ (2) $r > 0$
59. Is $X + Y < 1$? (1) $X < 8 / 9$ (2) $Y < 1 / 8$

60. If a and b are distinct integers and their product is not equal to zero, is $a > b$?
 (1) $(a^3b - b^3a)/(a^3b + b^3a - 2a^2b^2) < 0$ (2) $b < 0$
61. If m , r , x , and y are positive, is the ratio of m to r equal to the ratio of x to y ?
 (1) The ratio of m to y is equal to the ratio of x to r
 (2) The ratio of $m + x$ to $r + y$ is equal to the ratio of x to y
62. If $-2x > 3y$, is x negative? (1) $y > 0$ (2) $2x + 5y - 20 = 0$
63. If $a + b = 200$ and $a < b$, is $a + b > c + d$? (1) $c + d < 200$ (2) $b + c + d = 300$
64. If $0 < r < 1 < s < 2$, which one is less than 1
 I. r/s II. rs III. $s - r$
 I only II only III only I & II I & III
65. Did one of the three members of a certain team sell at least 2 raffle tickets yesterday?
 (1) The three members sold a total of 6 raffle tickets yesterday
 (2) No two of the members sold the same number of raffle tickets yesterday
66. If $z \times t < -3$, is $z < 4$? (1) $z < 9$ (2) $t < -4$
67. If a , b , c are integers such that $b > a$, is $b + c > a$? (1) $c > a$ (2) $abc > 0$
68. Is $|x - y| > |x| - |y|$? (1) $y < x$ (2) $xy < 0$
69. Is $x - y + 1$ greater than $x + y - 1$? (1) $x > 0$ (2) $y < 0$
70. If x and y are integers and $x > 0$, is $y > 0$? (1) $7x - 2y > 0$ (2) $-y < x$
71. Is z the median of any 3 positive integers x , y and z ? (1) $x < y + z$ (2) $y = z$
72. On the number line, the distance between x and y is greater than the distance between x and z . Does z lie between x and y on the number line?
 (1) $xyz < 0$ (2) $xy < 0$
73. One kilogram of a certain coffee brand consists of x kilograms of Type I coffee and y kilograms of Type II coffee. The cost of the brand is C dollars per kilogram, where $C = 6.5x + 8.5y$. Is $x < 0.8$?
 (1) $y \geq 0.15$ (2) $C \geq 7.30$
74. If $mv < pv < 0$, is $v > 0$? (1) $m < p$ (2) $m < 0$
75. If x is not equal to 0, then $\sqrt{[(x)^2]} / x = ?$
 -1 0 1 x $|x| / x$